

ESWEEK Tutorial

Sunday, 30th of September

Schedulability Analysis under Uncertainty using Formal Methods (part 2)

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LIPN, Université Paris 13, CNRS, France



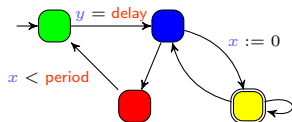
Outline

- 1 Parametric timed automata
- 2 IMITATOR in a nutshell
- 3 Modeling real-time systems with parametric timed automata
- 4 A case study: Verifying a real-time system under uncertainty

Model checking timed concurrent systems

■ Use formal methods

[Baier and Katoen, 2008]



A **model** of the system

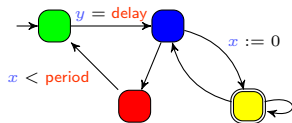
Red state is unreachable

A **property** to be satisfied

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■ Use formal methods

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is unreachable

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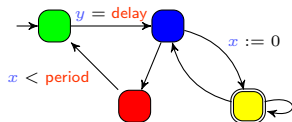
A **property** to be satisfied

■ Question: does the model of the system **satisfy** the property?

Model checking timed concurrent systems

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[Baier and Katoen, 2008]



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 is unreachable

A **model** of the system

A **property** to be satisfied

■ Question: does the model of the system **satisfy** the property?

Yes



No



Counterexample

Turing award (2007) to Edmund M. Clarke, Allen Emerson and Joseph Sifakis

Outline

1 Parametric timed automata

■ Timed automata

■ Parametric timed automata

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4 A case study: Verifying a real-time system under uncertainty

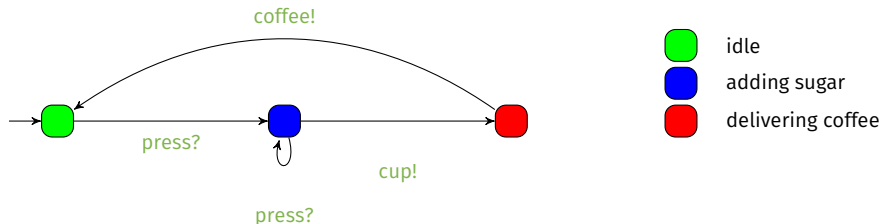
Timed automaton (TA)

- Finite state automaton (sets of locations)



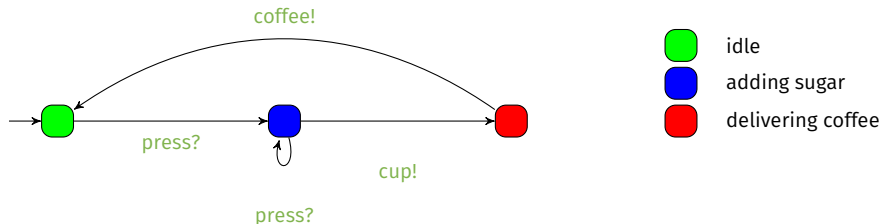
Timed automaton (TA)

- Finite state automaton (sets of locations and actions)



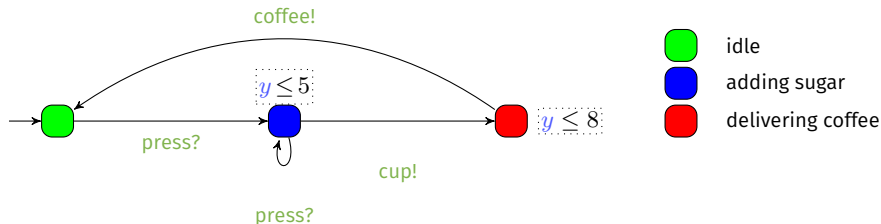
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 - Real-valued variables evolving linearly **at the same rate**



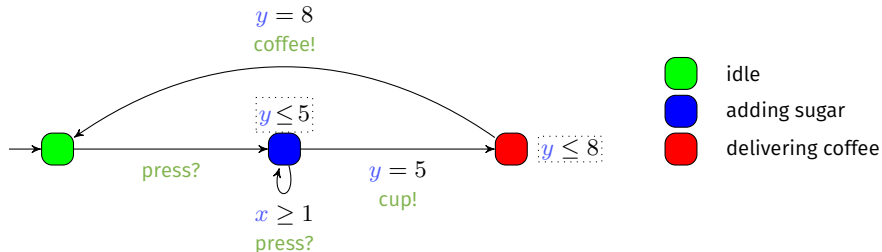
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 - Can be compared to integer constants in invariants
- Features
 - Location **invariant**: property to be verified to stay at a location



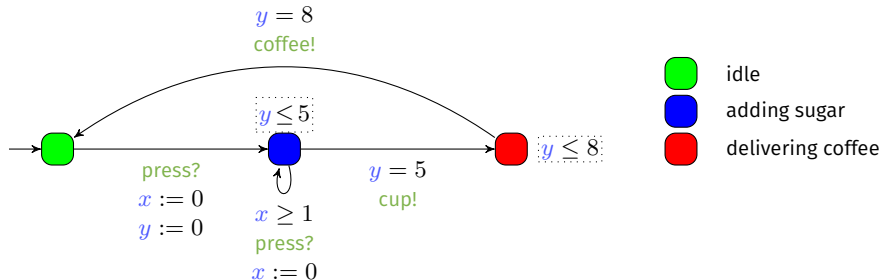
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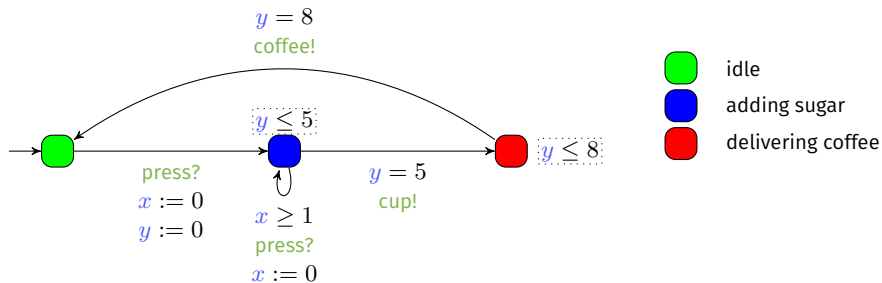


Timed automaton (TA)

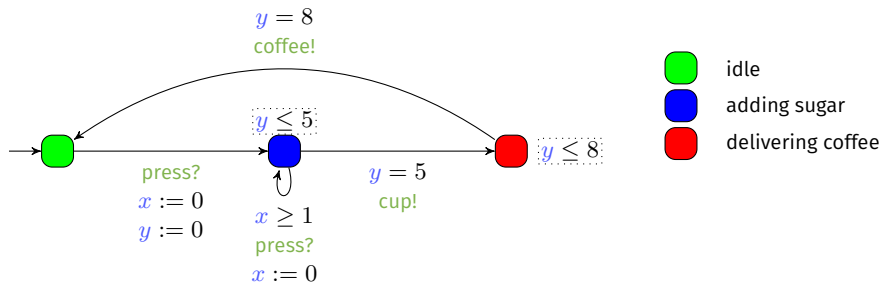
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- Features
 - Location **invariant**: property to be verified to stay at a location
 - Transition **guard**: property to be verified to enable a transition
 - Clock **reset**: some of the clocks can be **set to 0** along transitions



The most critical system: The coffee machine




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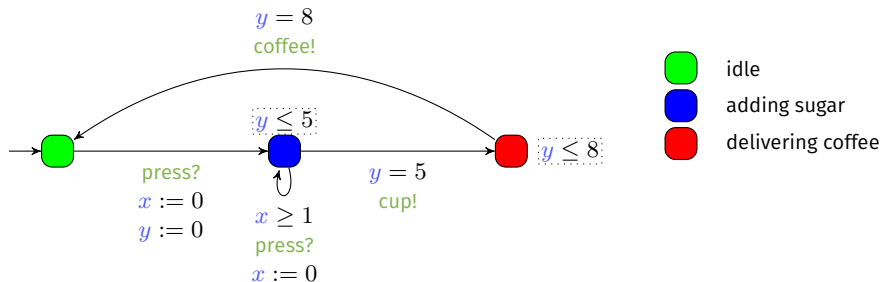


■ Example of concrete run for the coffee machine

■ Coffee with 2 doses of sugar

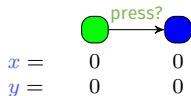

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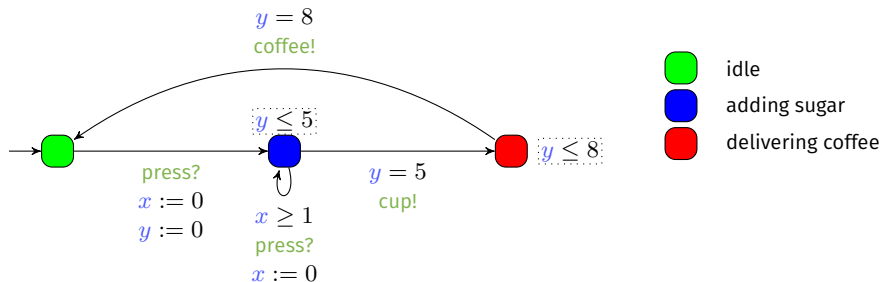


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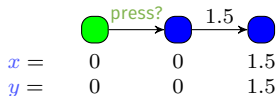


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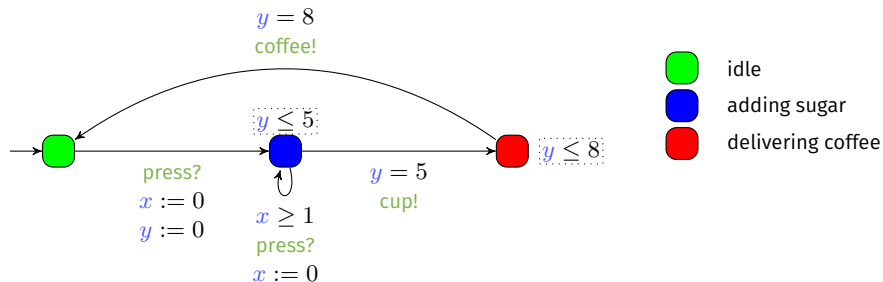


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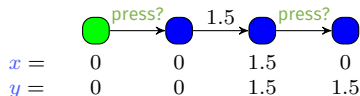


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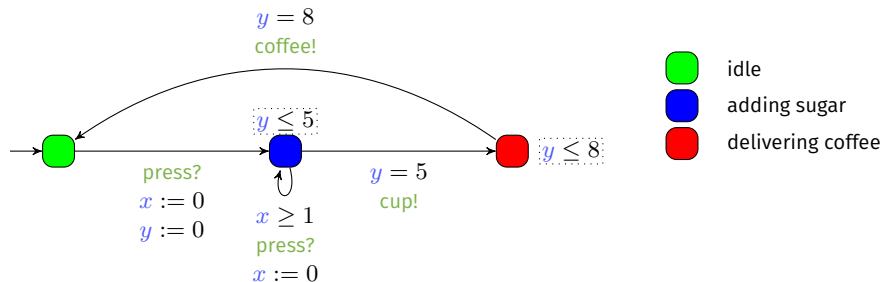


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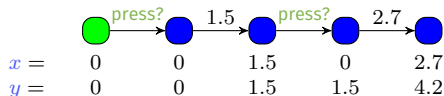


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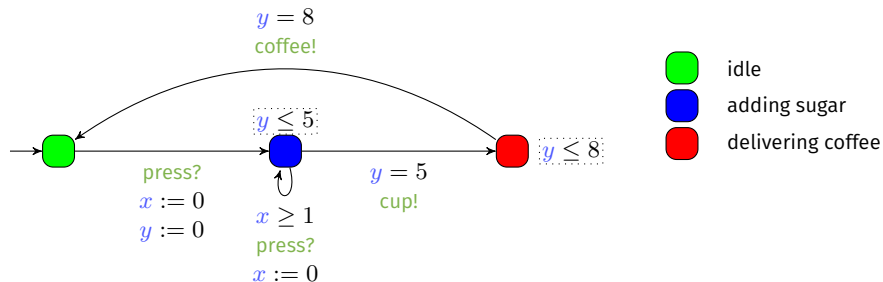


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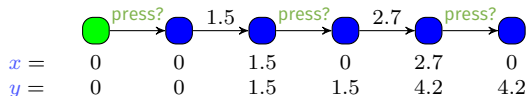


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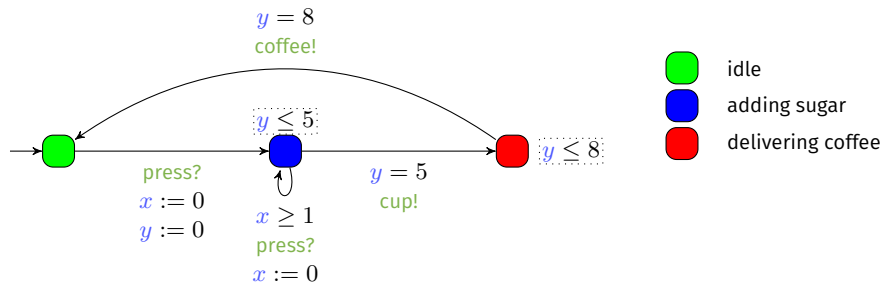


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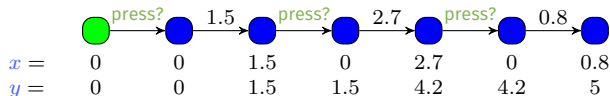


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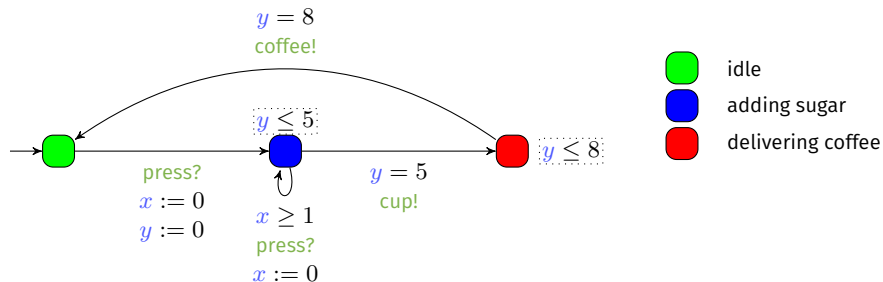


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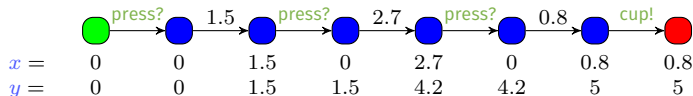


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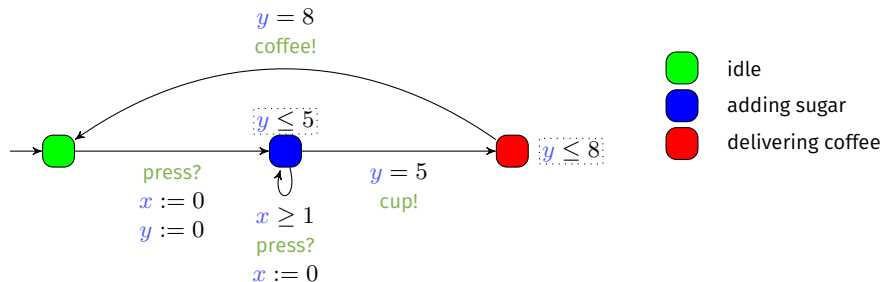


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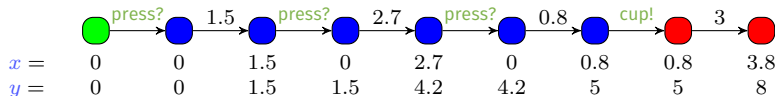


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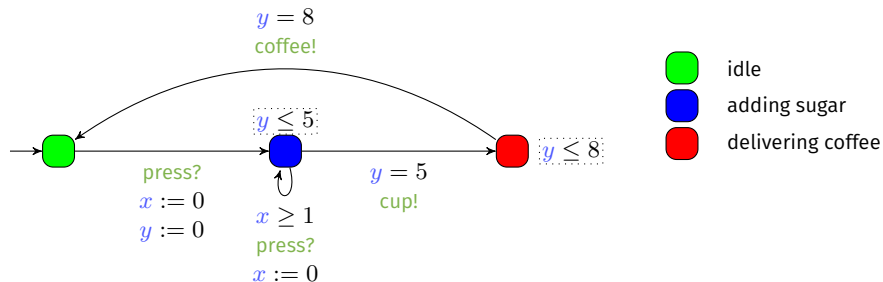


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Coffee with 2 doses of sugar

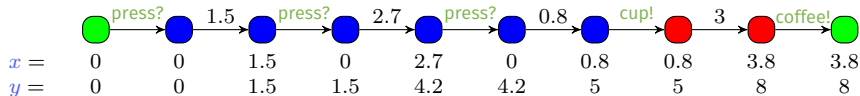


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Coffee with 2 doses of sugar



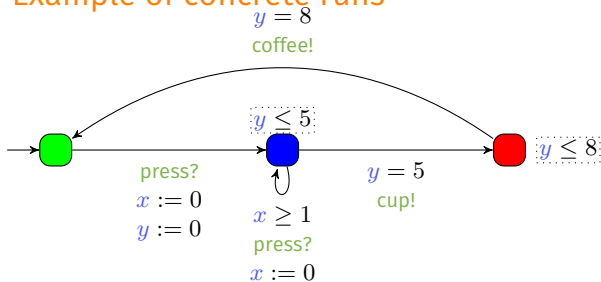
Concrete semantics of timed automata

- **Concrete state** of a TA: pair (l, w) , where
 - l is a **location**,
 - w is a **valuation** of each clock

Example:  $(\begin{smallmatrix} x=1.2 \\ y=3.7 \end{smallmatrix})$

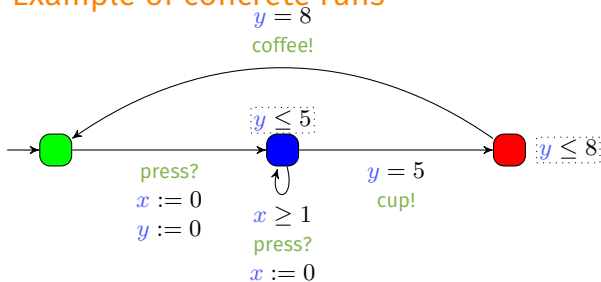
- **Concrete run**: alternating sequence of **concrete states** and **actions** or **time elapse**

Example of concrete runs



- Possible concrete runs for the coffee machine

Example of concrete runs



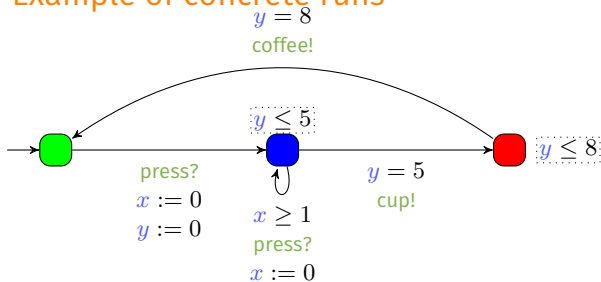
■ Possible concrete runs for the coffee machine

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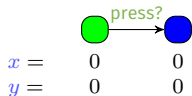
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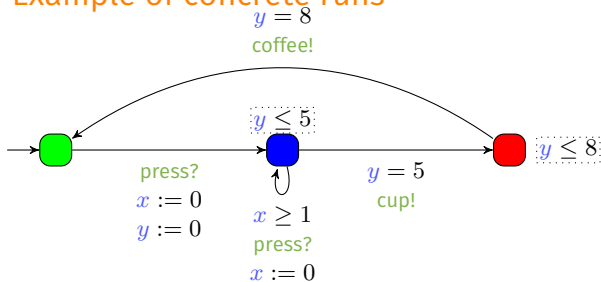


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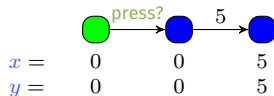


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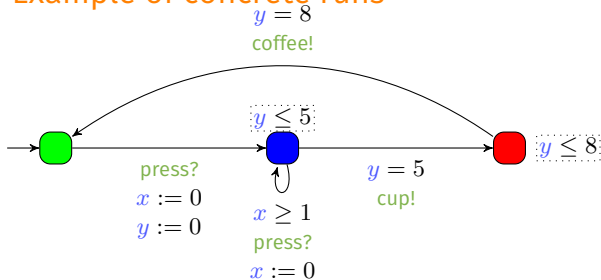


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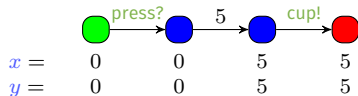


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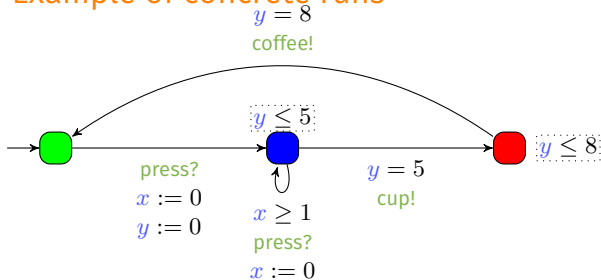


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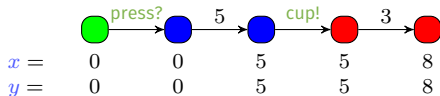


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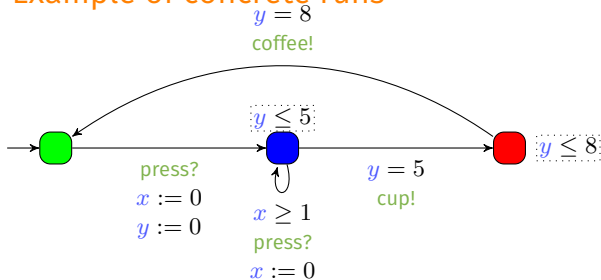


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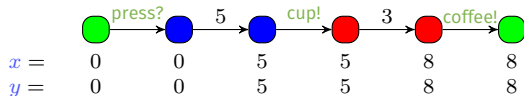


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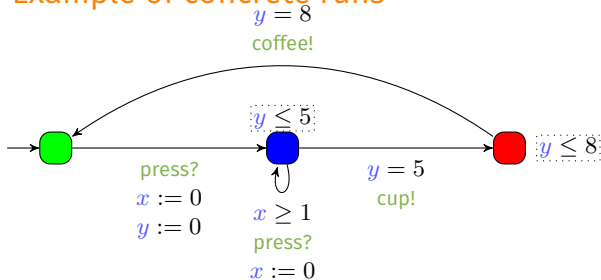


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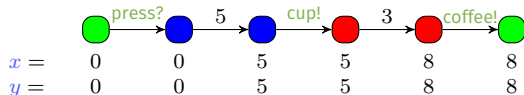


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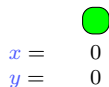


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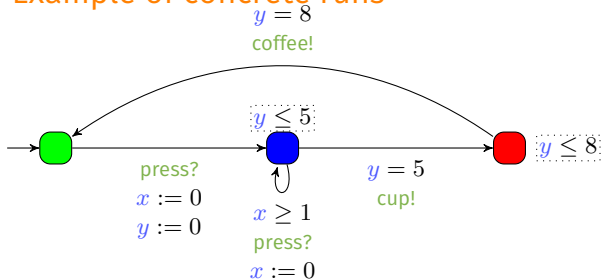
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■ Coffee with 2 doses of sugar

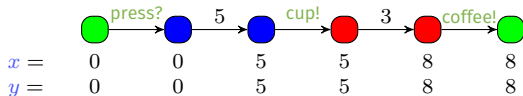


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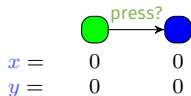


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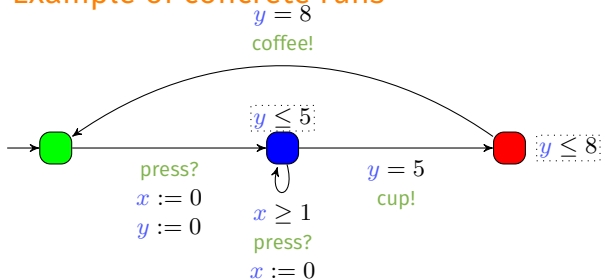
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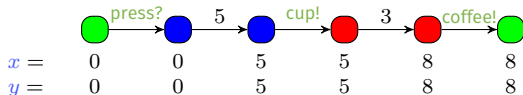


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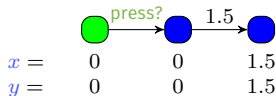


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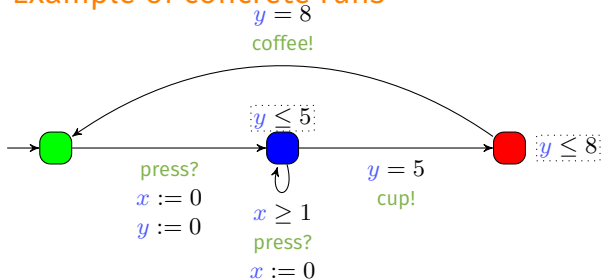
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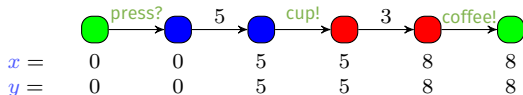


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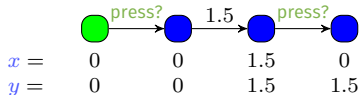


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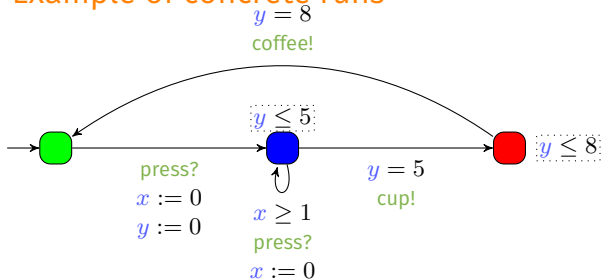
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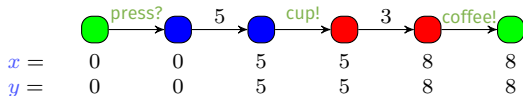


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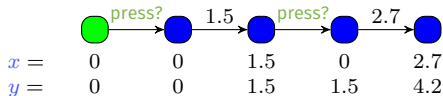


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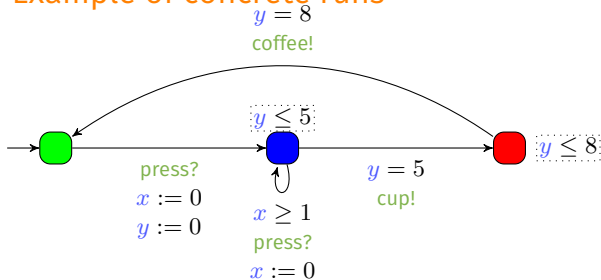
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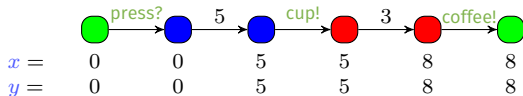


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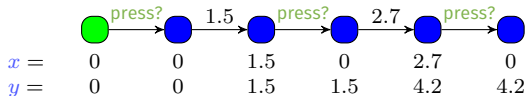


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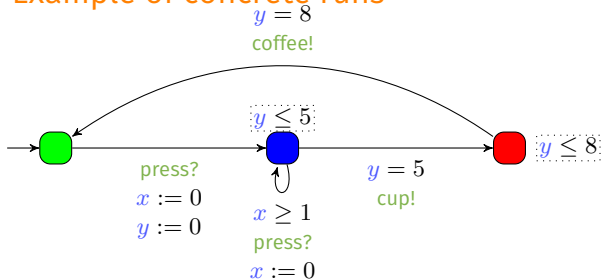
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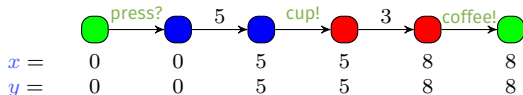


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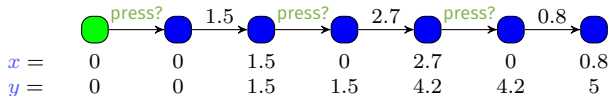


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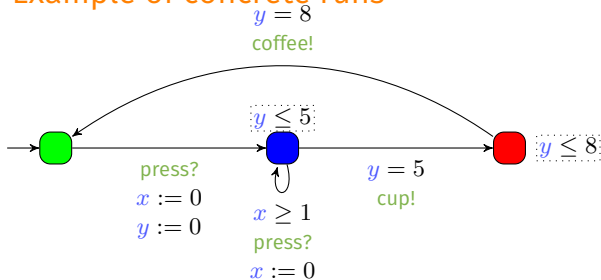
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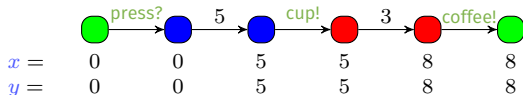


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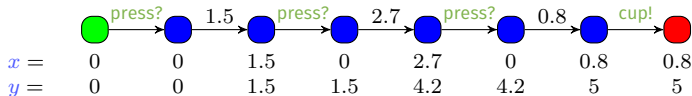


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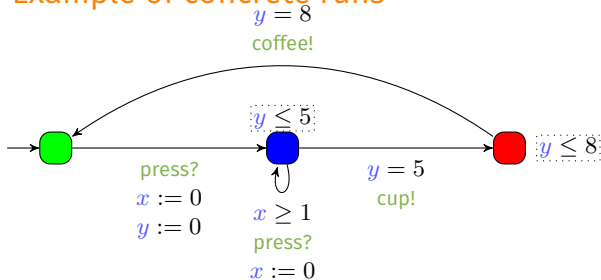
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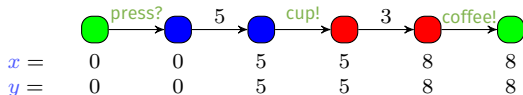


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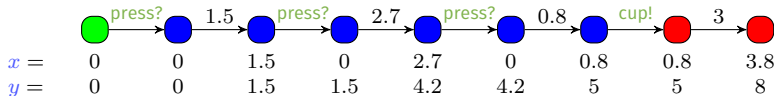


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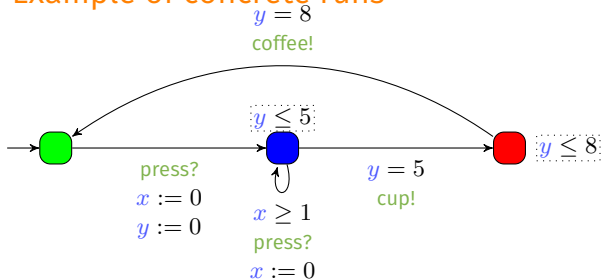
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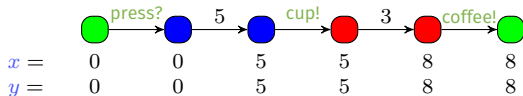


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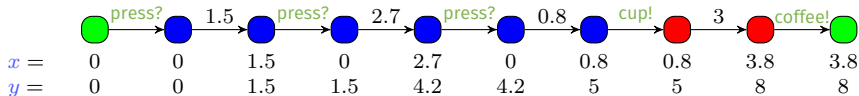


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Timed automata: A success story

- An expressive formalism

- Dense time
- Concurrency

- A tractable verification in theory

- Reachability is PSPACE-complete

[Alur and Dill, 1994]

- A very efficient verification in practice

- Symbolic verification: relatively insensitive to constants
- Several model checkers, notably UPPAAL
- Long list of successful case studies

[Larsen et al., 1997]

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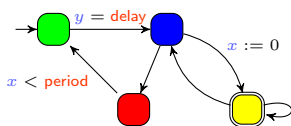
Beyond timed model checking: parameter synthesis

- Verification for **one** set of constants does not usually guarantee the correctness for other values
- Challenges
 - **Numerous verifications**: is the system correct for any value within $[40; 60]$?
 - **Optimization**: until what value can we increase 10?
 - **Robustness** [Markey, 2011]: What happens if 50 is implemented with 49.99?
 - **System incompletely specified**: Can I verify my system even if I don't know the period value with full certainty?

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 - **Optimization**: until what value can we increase 10?
 - **Robustness** [Markey, 2011]: What happens if 50 is implemented with 49.99?
 - **System incompletely specified**: Can I verify my system even if I don't know the period value with full certainty?
- **Parameter synthesis**
 - Consider that timing constants are unknown constants (**parameters**)

timed model checking



?

 is unreachable

A **property** to be satisfied

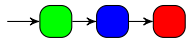
A **model** of the system

■ Question: does the model of the system satisfy the property?

Yes

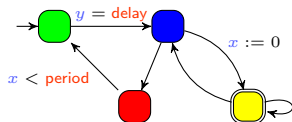


No



Counterexample

Parametric timed model checking



A **model** of the system

?

\models

 is unreachable

A **property** to be satisfied

- Question: **for what values of the parameters** does the model of the system **satisfy** the property?

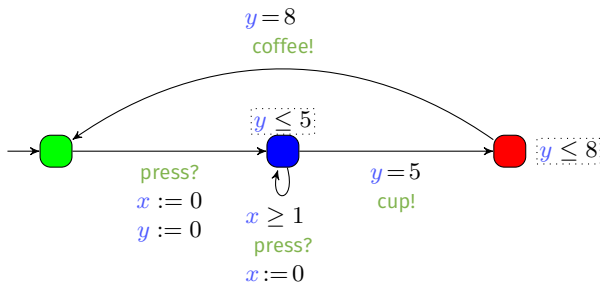
Yes if...



$$2\text{delay} > \text{period} \\ \wedge \text{period} < 20.46$$

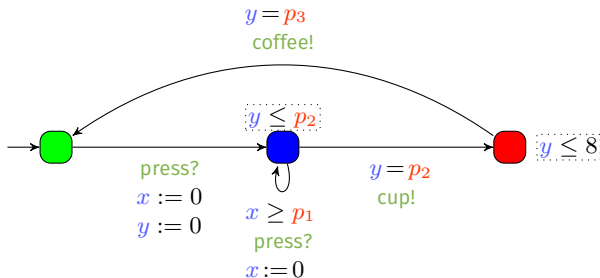
Parametric Timed Automaton (PTA)

- Timed automaton (sets of locations, actions and clocks)



Parametric Timed Automaton (PTA)

- Timed automaton (sets of locations, actions and clocks) augmented with a set P of parameters [Alur et al., 1993]
 - Unknown constants compared to a clock in guards and invariants

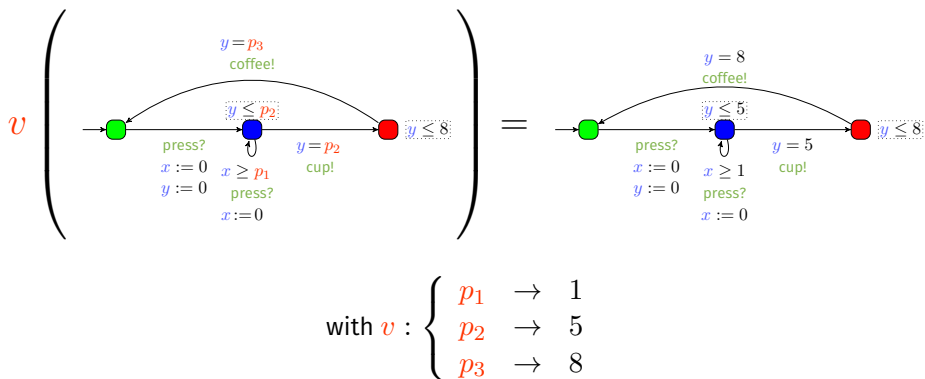


Notation: Valuation of a PTA

- Given a PTA \mathcal{A} and a parameter valuation v , we denote by $v(\mathcal{A})$ the (non-parametric) timed automaton where each parameter p is valued by $v(p)$

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Symbolic semantics of parametric timed automata

■ **Symbolic state** of a PTA: pair (l, C) , where

- l is a **location**,
- C is a convex polyhedron over X and P with a special form, called **parametric zone**

[Hune et al., 2002]

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Symbolic semantics of parametric timed automata

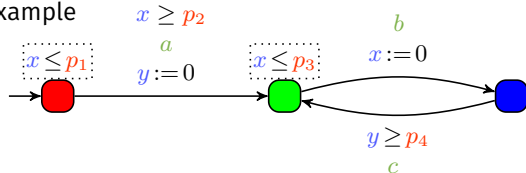
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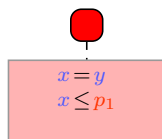
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■ **Symbolic run**: alternating sequence of **symbolic states** and **actions**

■ **Example**



■ Possible symbolic run for this PTA



Symbolic semantics of parametric timed automata

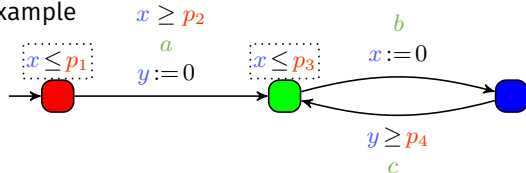
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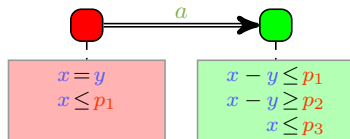
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Symbolic semantics of parametric timed automata

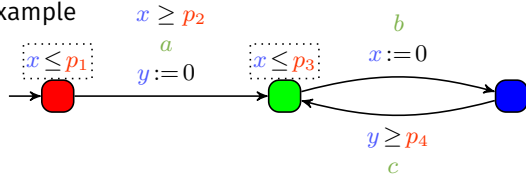
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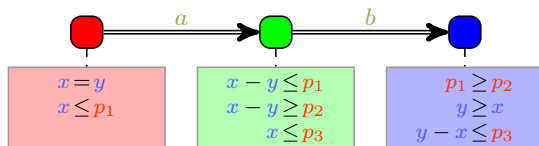
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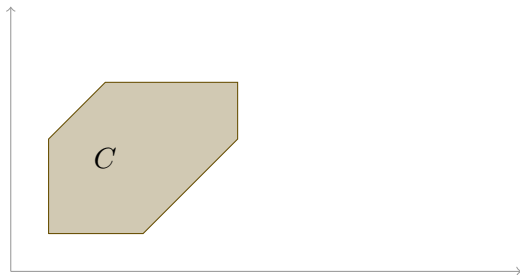


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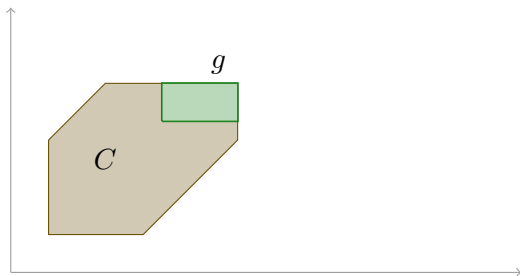
Symbolic semantics of PTA: Illustration

$$C' = [(C \cap g)]_{\textcolor{blue}{R}} \cap I(\textcolor{violet}{l}') \nearrow \cap I(\textcolor{violet}{l}')$$



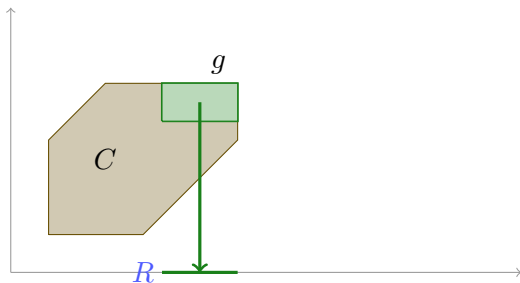
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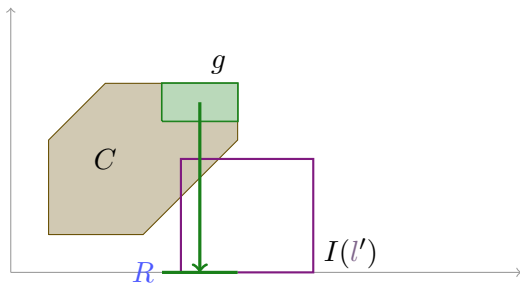
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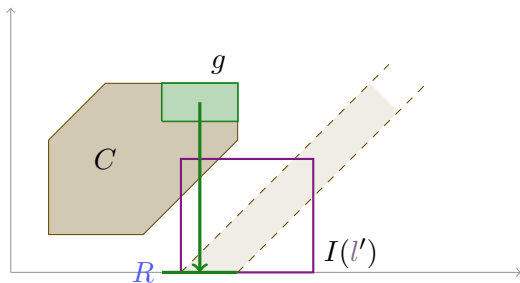
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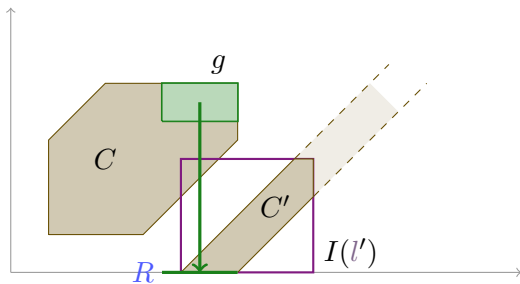
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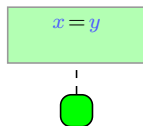
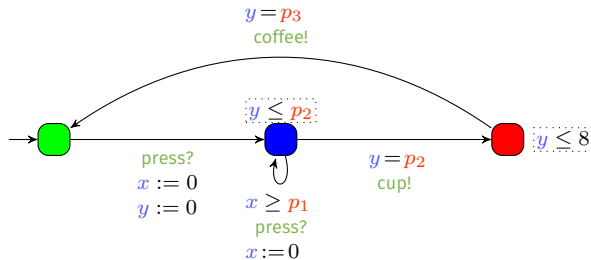


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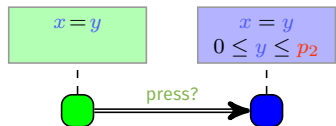
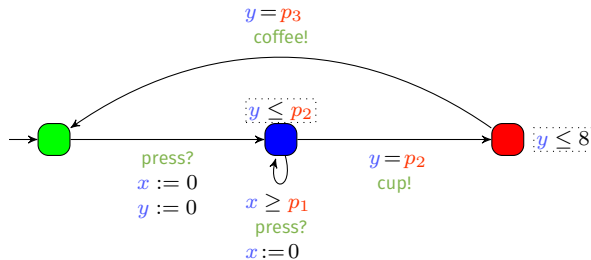
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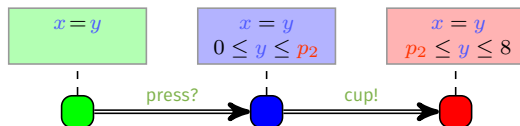
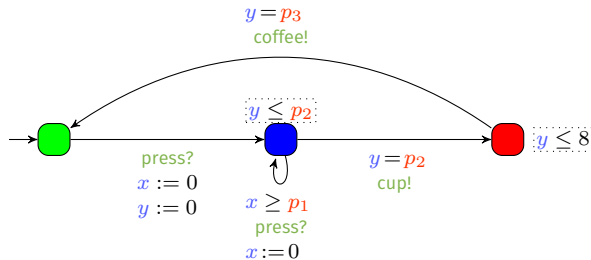
Symbolic exploration: Coffee machine



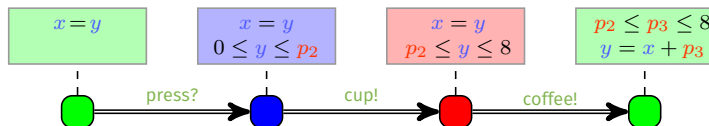
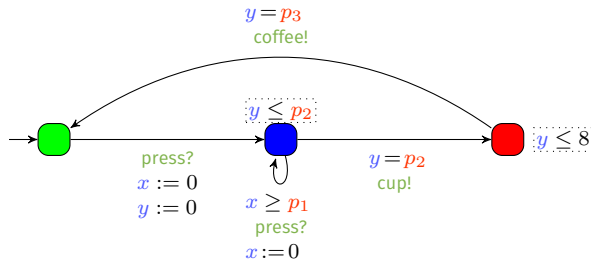
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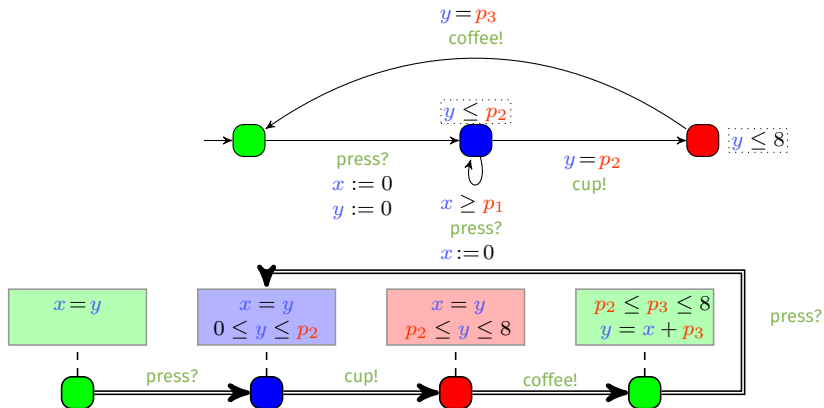
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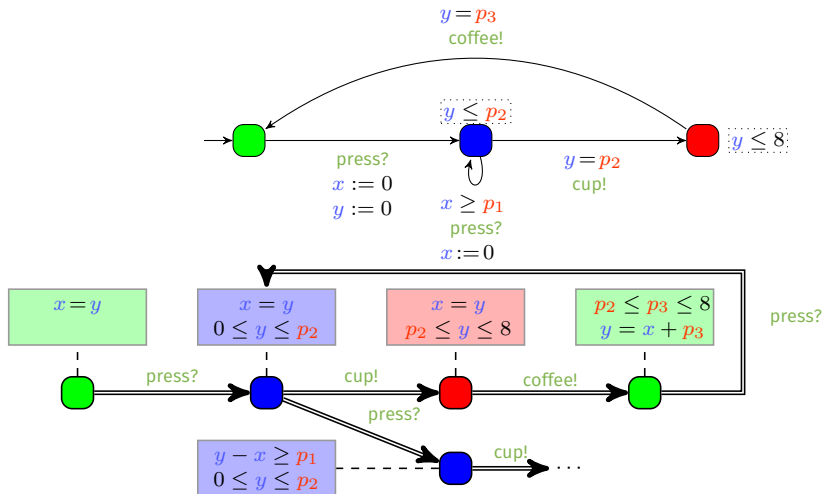
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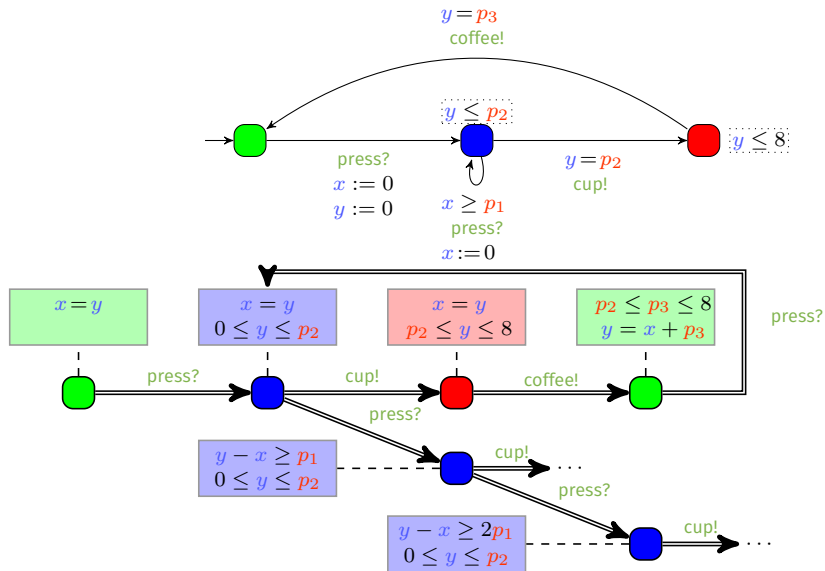
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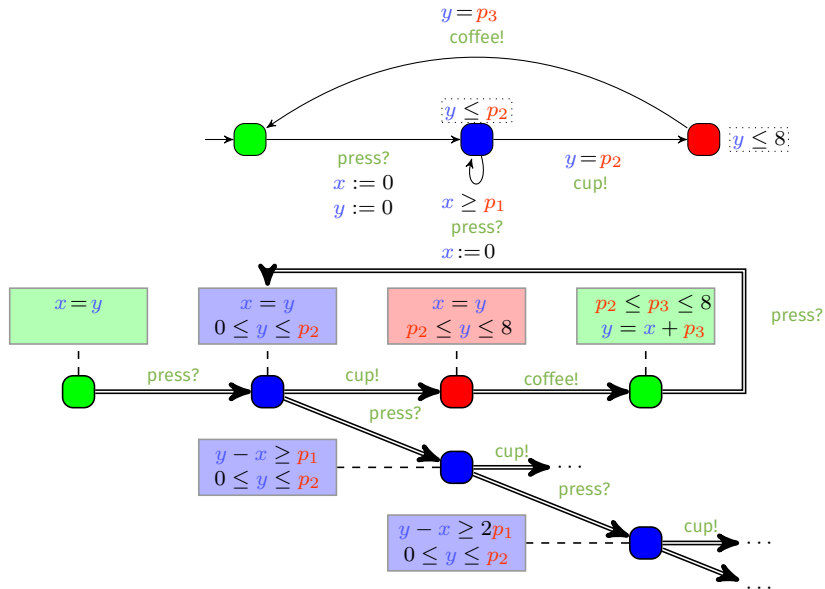
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Why studying decidability?

If a decision problem is **undecidable**, it is hopeless to look for algorithms yielding exact solutions (because that is **impossible**)

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If a decision problem is **undecidable**, it is hopeless to look for algorithms yielding exact solutions (because that is **impossible**)

However, one can:

- design **semi-algorithms**: if the algorithm halts, then its result is correct
- design algorithms yielding over- or under-**approximations**

Decision and computation problems for PTA

- **EF-Emptiness** “Is the set of parameter valuations for which a given location l is reachable empty?”

Example: “Does there exist at least one parameter valuation for which I can get a coffee with 2 sugars?”

- **EF-Universality** “Do all parameter valuations allow to reach a given location l ?”

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- No finite abstraction exists (unlike timed automata)

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Bad news

All interesting problems are undecidable for (general) parametric timed automata.

[ÉA, STTT 2017]

Undecidability in a nutshell

■ EF-emptiness problem

“Is the set of parameter valuations for which a given location l is reachable empty?”

[Alur et al., 1993, Miller, 2000, Doyen, 2007, Beneš et al., 2015]

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[Jovanović et al., 2015, André et al., 2016]

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[Jovanović et al., 2015, André et al., 2016]

■ Preservation of the untimed language

“Given a parameter valuation, does there exist another valuations with the same untimed language?”

[André and Markey, 2015]

Decidability in a nutshell

Reducing the number of clocks yields decidability of the EF-emptiness problem:

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- ✓ 1 parametric clock and arbitrarily many non-parametric clocks and integer-valued parameters [Beneš et al., 2015]
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Restraining the syntax brings decidability of some problems:

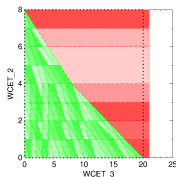
- L/U-PTAs [Hune et al., 2002, Bozzelli and La Torre, 2009, André and Markey, 2015, André and Lime, 2017, André et al., 2018b]
- PTAs with bounded integer-valued parameters [Jovanović et al., 2015]
- reset-PTAs [André et al., 2016, André et al., 2018c]

Outline

- 1 Parametric timed automata
- 2 IMITATOR in a nutshell**
- 3 Modeling real-time systems with parametric timed automata
- 4 A case study: Verifying a real-time system under uncertainty

IMITATOR

- A tool for modeling and verifying **timed concurrent systems** with unknown constants modeled with **parametric timed automata**
 - Communication through (strong) broadcast synchronization
 - Rational-valued shared discrete variables
 - **Stopwatches**, to model schedulability problems with preemption
- Synthesis algorithms
 - (non-Zeno) parametric model checking (using a subset of **TCTL**)
 - Language and trace preservation, and robustness analysis
 - Parametric deadlock-freeness checking



IMITATOR

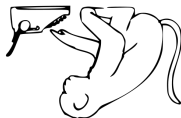
Under continuous development since 2008

[André et al., FM'12]

A library of benchmarks

- Communication protocols
- Schedulability problems
- Asynchronous circuits
- ...and more

Free and open source software: Available under the GNU-GPL license



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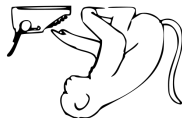
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Try it!

www.imitator.fr

Some success stories

- Modeled and verified an **asynchronous memory circuit** by ST-Microelectronics
- Parametric schedulability analysis of a prospective architecture for the flight control system of the **next generation of spacecrafts** designed at ASTRIUM Space Transportation [Fribourg et al., 2012]
- Verification of software product lines [Luthmann et al., 2017]
- Offline monitoring [ÉA, Hasuo, Waga @ ICECCS'18]
- Formal timing analysis of **music scores** [Fanchon and Jacquemard, 2013]
- Solution to a challenge related to a **distributed video processing system** by Thales

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Modeling real-time systems with timed automata

- Using timed automata

- [Abdeddaïm and Maler, 2001]

- Using stopwatch automata

- [Abdeddaïm and Maler, 2002]

- Using parametric timed automata

- [Cimatti et al., 2008]

- Using parametric stopwatch automata

- [Fribourg et al., 2012, Sun et al., 2013, Lipari et al., 2014]

- Using task automata

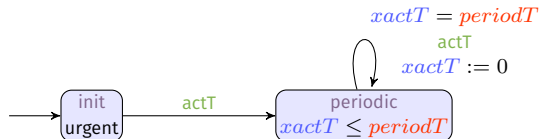
- [Norström et al., 1999, Fersman et al., 2007, André, 2017]

Modeling a periodic task T (exercise)

Periodic task T with period *period* T :

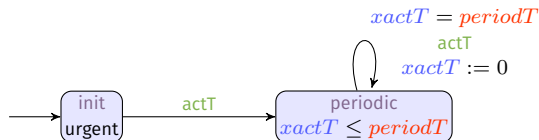
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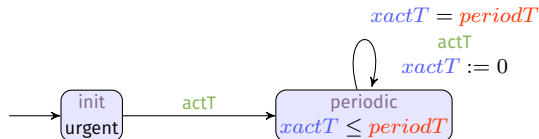
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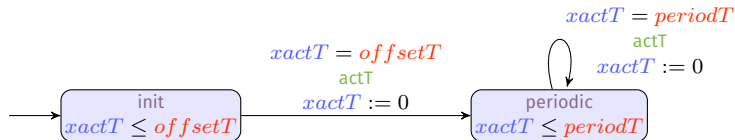
Periodic task T with period $periodT$ and $offsetT$:

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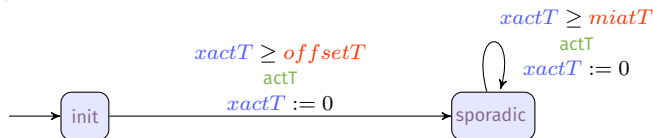


Modeling a sporadic task T (exercise)

Sporadic task T with minimum interarrival time $miatT$ and $offsetT$:

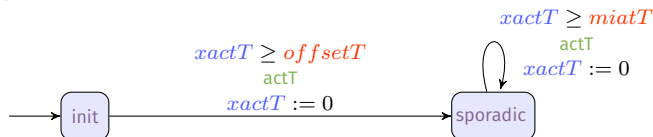
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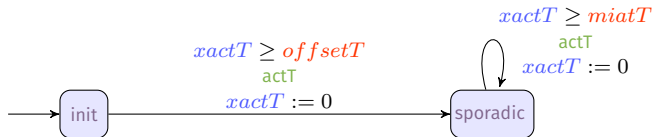


A more efficient modeling to avoid **clock divergence** in IMITATOR

- and hence optimize the computation

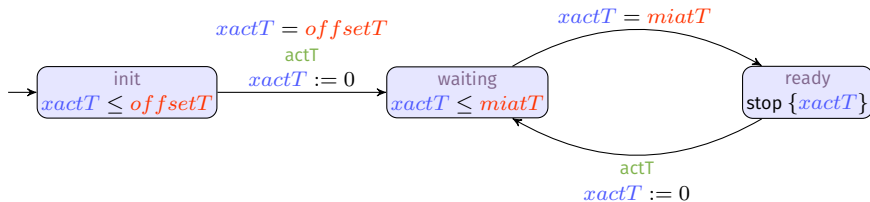
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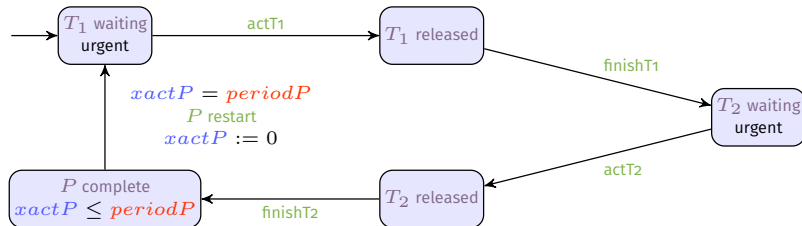
Trick: stop the computation of $xactT$ to avoid diverging

Modeling a task / pipeline

Pipeline P of two tasks T_1 and T_2
The pipeline has a period *period* P

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Pipeline P of two tasks T_1 and T_2
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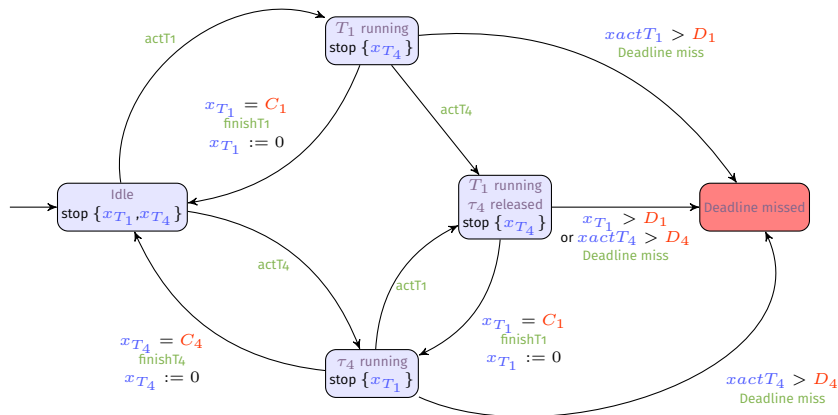


Modeling the preemptive fixed priority scheduler

A fixed-priority preemptive processor with two tasks T_1 and T_4

Timings for T_1 : period *period* T_1 , execution time period C_1 , deadline period D_1

■ and similarly for T_4



Outline

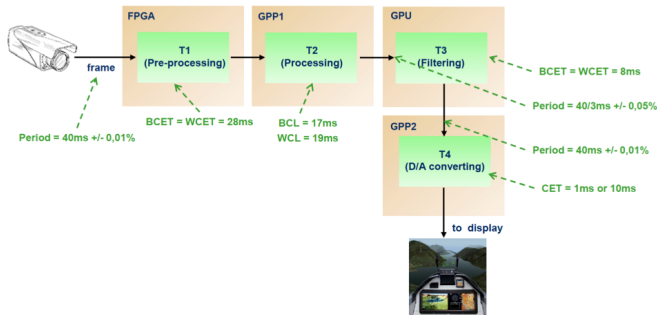
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The FMTV 2015 Challenge (1/2)

Challenge by Thales proposed during the WATERS 2014 workshop
Solutions presented at WATERS 2015

System: an unmanned aerial video system with **uncertain periods**

- Period constant but with a small uncertainty (typically 0.01 %)
- Not a jitter!



The FMTV 2015 Challenge (2/2)

Goal

Compute the end-to-end BCET and WCET times for a buffer size of $n = 1$ and $n = 3$

The FMTV 2015 Challenge (2/2)

Goal

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☹ Not a typical parameter synthesis problem?

- No parameters in the specification

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😊 A typical parameter synthesis problem

- The end-to-end time can be set as a **parameter**... to be synthesized
- The uncertain period is typically a **parameter** (with some constraint, e. g., $P1 \in [40 - 0.004, 40 + 0.004]$)

Methodology

- 1 Propose a PTA model with **parameters** for uncertain periods and the end-to-end time

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Note: not eliminating parameters allows one to know for **which values of the periods** the best / worst case execution times are obtained.

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- 6 Exhibit the minimum and the maximum

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To build the PTA model

■ Uncertainties in the system:

- $P1 \in [40 - 0.004, 40 + 0.004]$
- $P3 \in [\frac{40}{3} - \frac{1}{150}, \frac{40}{3} + \frac{1}{150}]$
- $P4 \in [40 - 0.004, 40 + 0.004]$

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- P1_uncertain
- P3_uncertain
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■ The end-to-end latency (another parameter): E2E

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■ Parameters:

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- P3_uncertain
- P4_uncertain

■ The end-to-end latency (another parameter): E2E

■ Others:

- the register between task 2 and task 3: discrete variable $\text{reg}_{2,3}$
- the buffer between task 3 and task 4: $n = 1$ or $n = 3$

Simplification

- T1 and T2 are synchronised; T1, T3 and T4 are asynchronised
 - (exact modeling of the system behaviour is too heavy)

Simplification

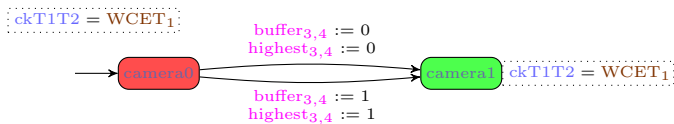
- T1 and T2 are synchronised; T1, T3 and T4 are asynchronised
 - (exact modeling of the system behaviour is too heavy)
- We choose a single arbitrary frame, called the **target** one
- We assume the system is initially in an arbitrary status
 - This is our only uncertain assumption (in other words, can the periods deviate from each other so as to yield any arbitrary deviation?)

The initialization automaton

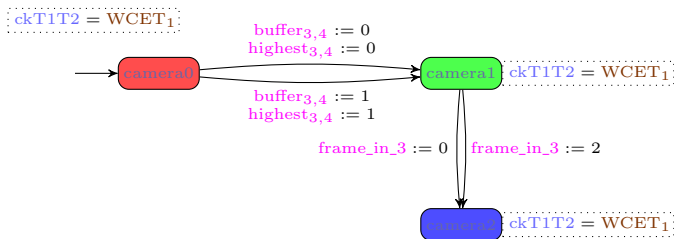
$$\text{ckT1T2} = \text{WCET}_1$$



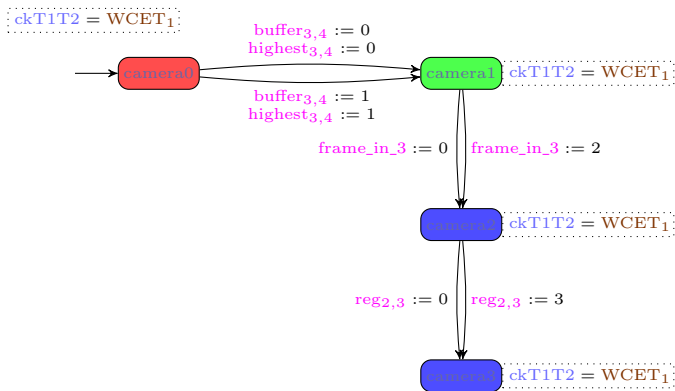
The initialization automaton



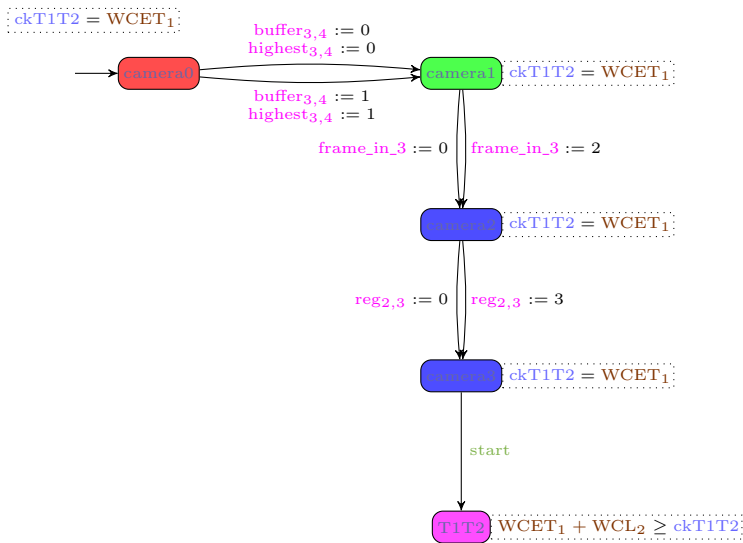
The initialization automaton



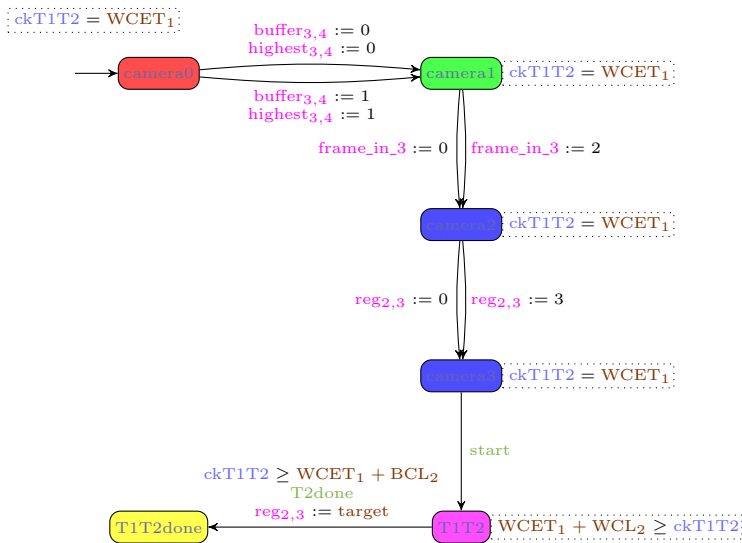
The initialization automaton



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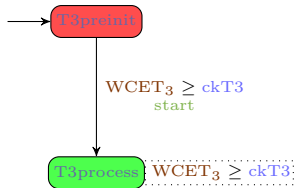
The initialization automaton



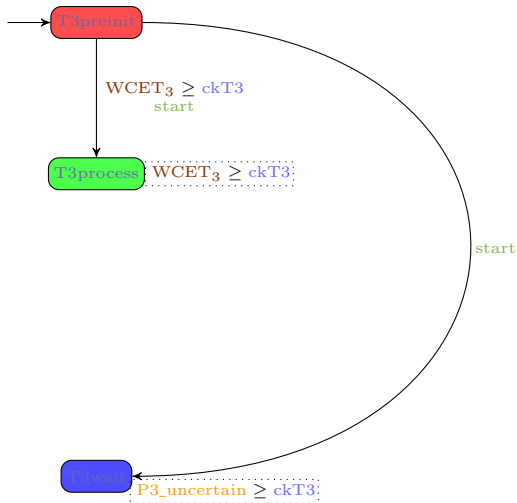
Task T3



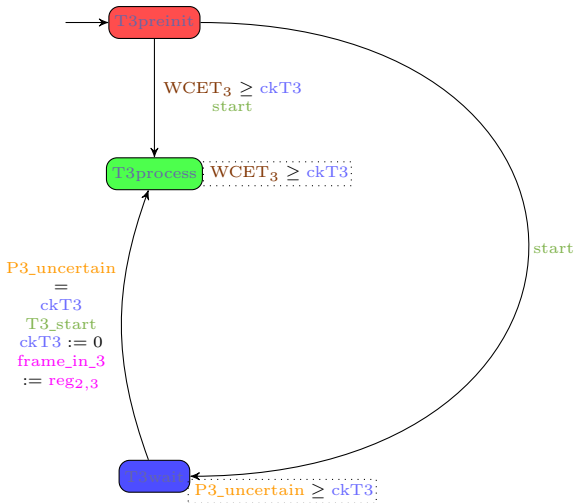
Task T₃



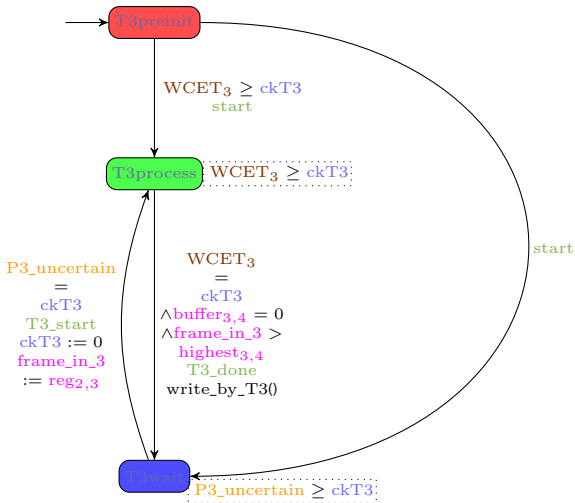
Task T3



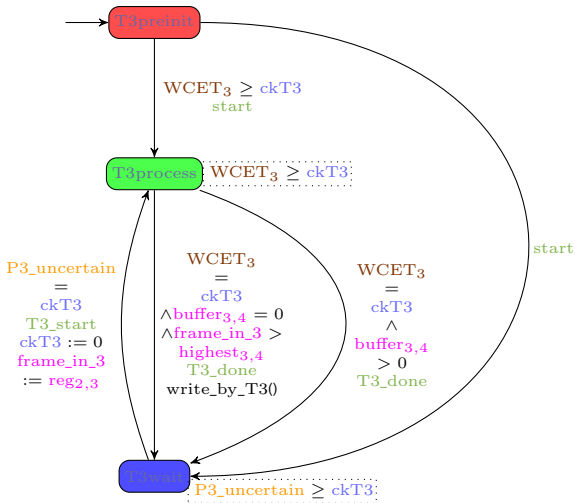
Task T3



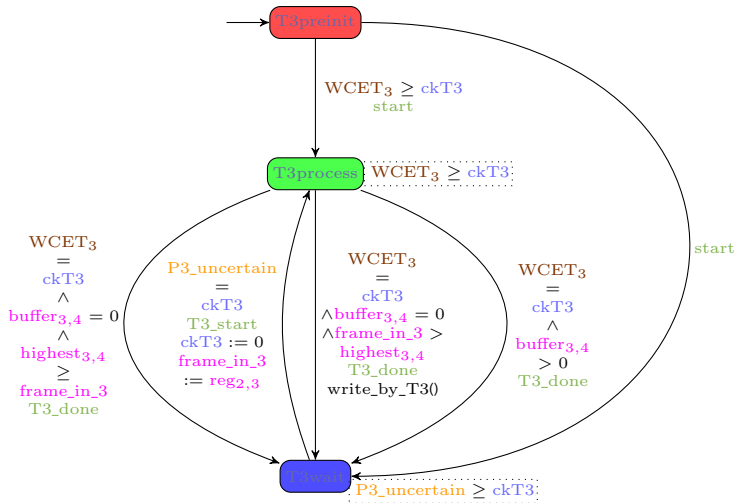
Task T3



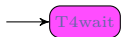
Task T3



Task T3

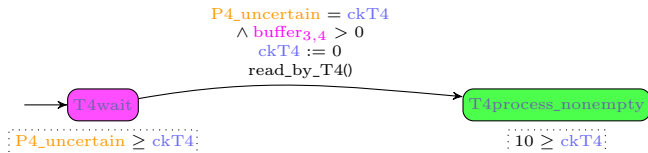


Task T₄

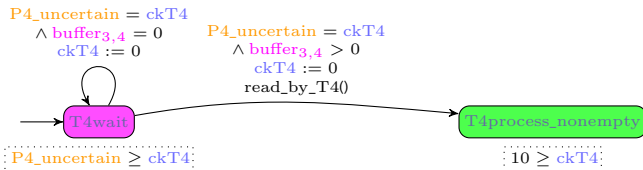


$P4_uncertain \geq ckT4$

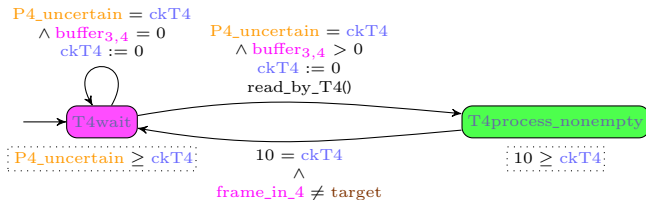
Task T4



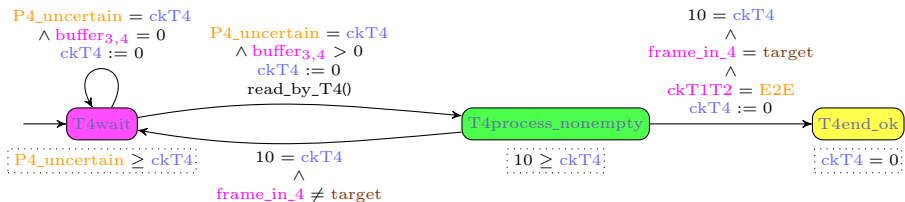
Task T4



Task T4



Task T4



Results

E2E latency results for $n = 1$ and $n = 3$

	$n = 1$	$n = 3$
min E2E	63 ms	63 ms
max E2E	145.008 ms	225.016 ms

Results obtained using IMITATOR in a few seconds

[ÉA, Lipari, Sun @ WATERS'15]

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Best paper award.

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